LAMBDA toolbox

Version 4.0 – User Manual (MATLAB)

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Table of Contents

Changelog

Date	Author(s)	Version	Comments
01/06/2024	Massarweh, L., Verhagen, S., Teunissen, P.J.G.	v1.0	User Manual (MATLAB) for LAMBDA 4.0 toolbox.
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List of Figures

List of Tables

Background Information

This document serves as 'User Manual' for the MATLAB implementation of LAMBDA 4.0 toolbox (Massarweh et al., 2024), which is distributed as free and open source software by the 'PNTLab' group (www.pntlab.tudelft.nl) at Delft University of Technology.

We refer readers to the official 'LAMBDA Documentation' for more mathematical details on the methodology and algorithms, along with the **Terms and Conditions**. In this short guide, we provide users with some information on how to run the software.

1. Getting Started

1.1. System Requirements

No additional MATLAB toolboxes are required by this LAMBDA 4.0 implementation, but we suggest referring to the MATLAB R2024a System Requirements for an optimal use of the toolbox functionalities. These requirements depend upon the platform in use:

Windows | https://mathworks.com/support/requirements/matlab-system-requirements.html

Mac | https://mathworks.com/support/requirements/matlab-mac.html

Linux | https://mathworks.com/support/requirements/matlab-linux.html

Note that these requirements might differ based on the MATLAB version in use; therefore they might also change in future releases of MATLAB software.

1.2. Installation Instructions

The MATLAB implementation of LAMBDA 4.0 toolbox does not require an installation, however we provide here a short guide for setting up your software:

I) **Download the Toolbox**: download the LAMBDA 4.0 toolbox from our official PNTLab's website at www.pntlab.tudelft.nl/LAMBDA.

II) **Extract the Files**: the ZIP archive named 'LAMBD4-master.zip' will be saved on your computer, and files can be extracted to your preferred directory.

III) **Add to MATLAB Path**: users can directly add this folder (and its sub-folders) to the MATLAB Path by selecting the "Home" tab, then clicking on "Set Path", including "Add with Subfolders" and saving this before closing. In this way, all the LAMBDA 4.0 functionalities are now ready to be used.

Alternatively to Step (III), you could simply use `addpath('LAMBDA_toolbox')` if you do not want to add permanently this toolbox to your Path. With this command you will be still able to access to all functionalities during your current MATLAB session.

In the README.txt file (see Figure 1) users will find a complete list of all documents and functions, along with some relevant literature within the folder "LAMBDA_papers".

```
The Least-squares AMBiguity Decorrelation Adjustment method,      |
LAMBDA toolbox, Version v4.0, MATLAB implementation.              |
------------------------------------------------------------------|
Copyright: Geoscience & Remote Sensing department @ TUDelft       |
Contact email:  LAMBDAtoolbox-CITG-GRS@tudelft.nl                 |
------------------------------------------------------------------/
Last update: 01-06-2024
________________________________________________________________________________
1 - Main MATLAB routines:

        LAMBDA.m                        | Replaces    LAMBDA 3.0 (2012)
        Ps_LAMBDA.m                     | Replaces Ps-LAMBDA 1.0 (2013)
________________________________________________________________________________
2 - Main LAMBDA documents:

        LAMBDA - Documentation.pdf      | LAMBDA 4.0 official documentation
        LAMBDA - UM for MATLAB.pdf      | LAMBDA 4.0 user manual (MATLAB)
________________________________________________________________________________
3 - Main LAMBDA functionalities:

        Accessible by using << addpath('LAMBDA_toolbox') >>

> Decorrelation
        decorrelateVC.m                 | Used in LAMBDA.m / Ps_LAMBDA.m
        decomposeLtDL.m                 | Used in LAMBDA.m / Ps_LAMBDA.m
        transformZ.m                    | Used in LAMBDA.m / Ps_LAMBDA.m

> Estimation
        estimatorIR.m                 |1| Used in LAMBDA.m / Ps_LAMBDA.m
        estimatorIB.m                 |2| Used in LAMBDA.m / Ps_LAMBDA.m
        estimatorILS.m                |3| Used in LAMBDA.m / Ps_LAMBDA.m
        estimatorILS_enum.m           |4| Used in LAMBDA.m
        estimatorPAR.m                |5| Used in LAMBDA.m
        estimatorVIB.m                |6| Used in LAMBDA.m / Ps_LAMBDA.m
        estimatorIA_FFRT.m            |7| Used in LAMBDA.m
        estimatorIAB.m                |8| Used in LAMBDA.m
        estimatorBIE.m                |9| Used in LAMBDA.m

> Evaluation
        computeSR_IBexact.m           |1| Used in LAMBDA.m / Ps_LAMBDA.m
        computeSR_ADOPapprox.m        |2| Used in          / Ps_LAMBDA.m
        computeSR_LB_Variance.m       |3| Used in          / Ps_LAMBDA.m
        computeSR_UB_ADOP.m           |4| Used in          / Ps_LAMBDA.m
        computeSR_LB_Eigenvalue.m     |5| Used in          / Ps_LAMBDA.m
        computeSR_UB_Eigenvalue.m     |6| Used in          / Ps_LAMBDA.m
        computeSR_LB_Pullin.m         |7| Used in          / Ps_LAMBDA.m
        computeSR_UB_Pullin.m         |8| Used in          / Ps_LAMBDA.m
        computeSR_Numerical.m         |9| Used in          / Ps_LAMBDA.m

> Auxiliary
        checkMainInputs.m               | Used in LAMBDA.m / Ps_LAMBDA.m
        computeADOP.m                   | Used in          / Ps_LAMBDA.m
        computeFFRTcoeff.m              | Needed for estimatorIA_FFRT.m
        computeIGT_row.m                | Needed for transformZ.m
        computeInitialEllipsoid.m       | Needed for estimatorILS_enum.m
        computeNumSamples.m             | Needed for computeSR_Numerical.m
________________________________________________________________________________
4 - Main MATLAB examples:

        Accessible in the folder 'LAMBDA_examples'

        MODEL_GeometryFree.m            | Generates GNSS geometry-free models
        RUN_example_1.m                 | Example #1 for the LAMBDA 4.0 toolbox
        RUN_example_2.m                 | Example #2 for the LAMBDA 4.0 toolbox
        RUN_example_3.m                 | Example #3 for the LAMBDA 4.0 toolbox
        RUN_example_X.m                 | Template for creating new examples
________________________________________________________________________________
5 - Main LAMBDA literature:

        Provided in the folder 'LAMBDA_papers' in PDF
________________________________________________________________________________|
```

Figure 1: An overview of the LAMBDA 4.0 software package content (for MATLAB).

1.3. Software Package Content

The LAMBDA 4.0 toolbox software package for this MATLAB implementation includes the files and folders that have been shown in Figure 2.

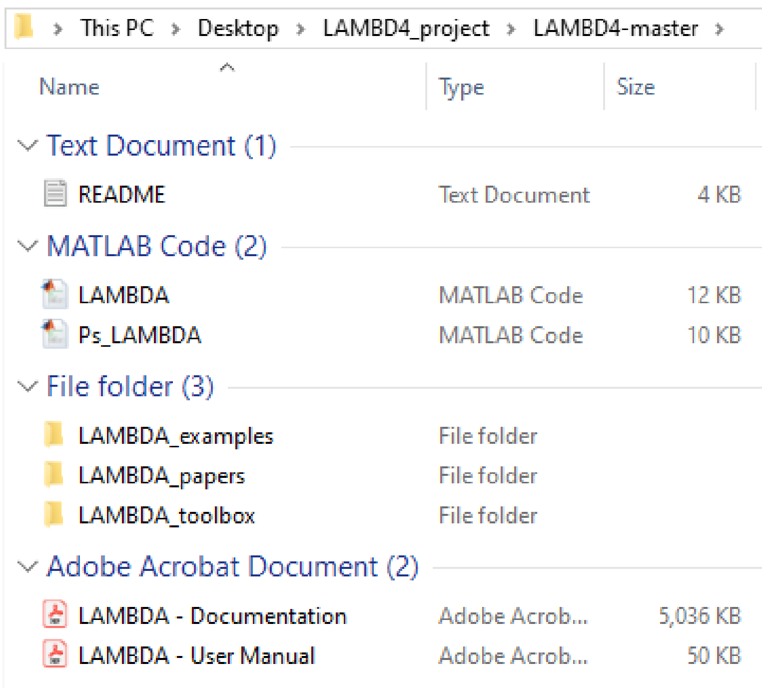

Figure 2: List of files/folder in LAMBDA 4.0 software package (for MATLAB)

A short description of these files/folders is provided in the following Table 1.

Table 1: List of files/folders included in LAMBDA 4.0 software package (for MATLAB)

File/Folder	Description
README.txt	Text file with content overview
LAMBDA	Main script with **LAMBDA** functionalities
Ps_LAMBDA	Main script with **Ps-LAMBDA** functionalities
\ LAMBDA_toolbox	Folder with all toolbox functionalities
\ LAMBDA_examples	Folder with some introductory examples
\ LAMBDA_papers	Folder with some relevant publications
LAMBDA– Documentation.pdf	PDF with main mathematical descriptions
LAMBDA– User Manual.pdf	PDF with the toolbox guide (MATLAB)

2. Run Some Examples

Once the LAMBDA 4.0 toolbox main scripts and functionalities are accessible within your MATLAB environment, you can start making use of all routines, or just try to get a first experience with the examples in the next sections. These illustrative examples might then be changed in future updates, as well as for this User Manual.

In general, users can simply make use of '`LAMBDA.m`' and '`Ps_LAMBDA.m`' main scripts, or directly operate with the standalone functions of this v4.0 toolbox. Each function has a short description of inputs and outputs, along with a list of possible dependencies (from other functions), if relevant. Additional information is available in the functions' header, which can be accessed directly by the command: `help nameFunction`.

Let's now show a few illustrative examples.

2.1. **Example #1**: on LAMBDA functionalities

In `RUN_example_1.m`, we make use of a GNSS geometry-free model given N receivers tracking M satellites on a total of J frequencies over K epochs. This model can be easily generated by the user using the available function `MODEL_GeometryFree.m`, where both GPS and Galileo signal frequencies can be selected. This MATLAB function returns the full variance-covariance matrix for float ambiguities and parameters, later used to generate one single float sample processed by LAMBDA with different estimators.

For this illustrative example[1], we use $K = 1$ (single epoch), while assuming a network of $N = 5$ receivers tracking $M = 20$ GPS satellites on $J = 3$ frequencies, i.e., L1/L2/L5. The standard deviation for the undifferenced code and phase observations is 30cm and 3mm, respectively, while an ionosphere-fixed model is assumed. Different scenarios can still be tested by the users, as well as for more specific LAMBDA configurations.

The main LAMBDA function call is

```
[a_fix,sqnorm,nFixed,SR,Z_mat,Qz_hat] = LAMBDA(a_hat,Qa_hat,iMethod);
```

where `a_hat` and `Qa_hat` are respectively a float ambiguity vector and its vc-matrix, while `iMethod` refers to the estimator made available in LAMBDA script. More details on the inputs and outputs can be found using the command `help LAMBDA`.

[1] The only purpose of this example is to show how to run different LAMBDA estimators given a certain mixed-integer estimation problem. Note that BIE is excluded in this example since dealing with a very large number of ambiguity components, i.e., around 230, thus having a much larger computational time.

The result of this script is given in Figure 3. Note that the CPU time [s] for each estimators might be different on your machine, while the other quantities should be exactly the same as shown below in the MATLAB Command Window.

```
-----------------------------------------------------------------
EXAMPLE #1 - Summary results for LAMBDA:
-----------------------------------------------------------------
                                Method_list    nn_vect    nFixed_list    sqnorm_list    timeCPU_list
                                ___________    _______    ___________    ___________    ____________

    Float                            0           228           0                0           0.229
    Integer Rounding                 1           228         228           338.63           0.212
    Integer Bootstrapping            2           228         228           193.49           0.237
    ILS (search-and-shrink)          3           228         228           193.49           0.196
    ILS (enumeration)                4           228         228           193.49           0.337
    PAR ( >99.0% )                   5           228         152           126.84           0.193
    VIB-ILS (2 blocks)               6           228         228           193.49           0.193
    IA-FFRT ( <0.1% )                7           228           0                0           0.212
    IAB ( beta=0.5 )                 8           228           0                0             0.2

> Number of real-valued parameters = 76
> Number of integer ambiguities = 228
> IB success rate = 91.61% (after decorrelation)
>> |
```

Figure 3: Command Window output from Example #1, see text for more information.

2.2. **Example #2**: on Ps-LAMBDA functionalities

In `RUN_example_2.m`, we consider an illustrative 2D ambiguity problem as

$$D = \begin{bmatrix} 0.14^2 & \\ & 0.30^2 \end{bmatrix}, \qquad L = \begin{bmatrix} 1 & 0 \\ +1.499 & 1 \end{bmatrix}$$

with $D \in \mathbb{R}^2, L \in \mathbb{R}^2$ being the diagonal and lower unitriangular matrices from the $\mathrm{L^TDL}$-decomposition of the variance-covariance matrix of float ambiguities. Hence, we make use of different Ps-LAMBDA functionalities in order to compute bounds or approximations for the success rate, while accounting or not for the ambiguity decorrelation.

More details on inputs/outputs and configurations can be found using the command `help Ps_LAMBDA`. For the numerical simulation, we have considered a bootstrapping estimator to compare its success rate against the exact one computed in `computeSR_IBexact`. As visible in Figure 4, we obtain SR equal to 90.44% and 90.41% respectively for the former and the latter case. After the decorrelation, the diagonal entries in the matrix D are both around 0.205^2 while the off-diagonal term in L is reduced to $+0.069$. In this way the SR_z (for decorrelated ambiguities) is largely improved to around 97.1%,

```
-------------------------------------------------------------------
EXAMPLE #2 - Summary results for Ps_LAMBDA:
-------------------------------------------------------------------
                    Method_list    SR_a     SR_z
                    ___________    _____    _____

    IB_exact             1         90.41    97.08
    ADOP_approx          2         97.08    97.08
    LB_Variance          3         64.36    97.06
    UB_ADOP              4         97.74    97.74
    LB_Eigenvalue        5         40.18    96.35
    UB_Eigenvalue        6           100     97.7
    LB_Pullin            7          94.9     94.9
    UB_Pullin            8         97.13    97.13
    IB_numerical         9         90.44    97.09

LEGEND:
SR_a = Success Rate in the original ambiguity parametrization
SR_z = Success Rate in the decorrelated ambiguity parametrization
>>
```

Figure 4: Command Window output from Example #2, see text for more information.

2.3. **Example #3**: comparison BIE and ILS solutions

In `RUN_example_3.m`, we consider a (short) single-baseline GPS geometry-based model with 6 tracked satellites on L1. For undifferenced code/phase measurements, we assume a 20cm/2mm standard deviation with an elevation weighting, and we compute the variance-covariance matrix for the ambiguities and parameters. After the decorrelation, we still have a low success rate of 30.7%, which is not sufficient for a successful ambiguity resolution, so we opt for the Best Integer Equivariant (BIE) solution. The latter is compared – in terms of Mean Squared Error (MSE) – against both the Float and Fixed (ILS) solutions that are computed using 10’000 samples (synthetically generated in the script).

The results are shown in Figure 5, where the MSE ratio (with respect to the Float) is shown for the different solutions. Even if BIE has the largest computational time (on the 10’000 samples), it also provides the minimum MSE value for *b*aseline parameters, which refer here to East-North-Up components[2]. On the other hand, the ILS solution has worse MSE performance than the float solution, given that in many instances ambiguities are wrongly fixed.

[2] Same results hold also with a different parametrization, e.g., using Cartesian coordinates.

```
> SOLUTIONS:
- Float done (istantaneous)
- ILS done in 0.088 [s]
- BIE done in 3.241 [s]
--------------------------------------------------------------
EXAMPLE #3 - Comparison of Mean Squared Error (MSE) ratio:
--------------------------------------------------------------
                    MSE_b_ratios
                    ____________

    Float/Float           1
    ILS/Float           1.2
    BIE/Float         0.893

> Number of real-valued parameters = 3
> Number of integer ambiguities = 5
> IB success rate = 30.70% (after decorrelation)
>> |
```

Figure 5: Command Window output from Example #3, see text for more information.

For the sake of completeness, horizontal errors from this positioning example are shown in Figure 6, where we consider the 10’000 float samples adopted for the MSE simulations.

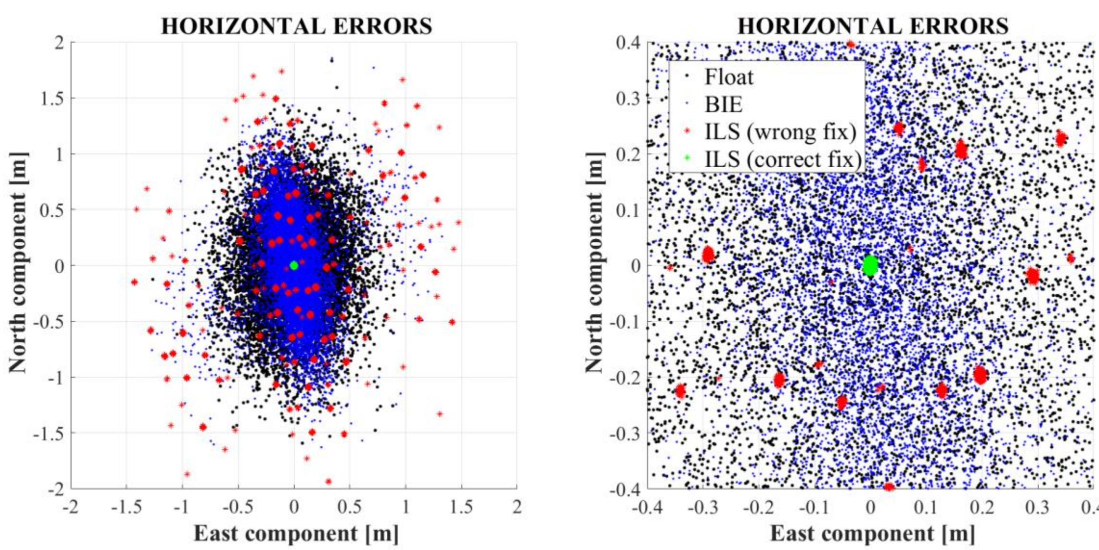

Figure 6: Graphical example of the horizontal errors for the Example #3 using different estimators, where the 10’000 float samples are shown in black. The red and green symbols refer to the fixed parameters with a wrong and a correct ambiguity resolution, while the BIE solution is shown in blue color. The latter is optimal in the minimum MSE sense.

2.4. **Example #X**: "template"

Other numerical examples will be added in future updates of this software packages, and its User Manual. A MATLAB template has been provided to users to more easily construct additional examples. We encourage LAMBDA users to contact us in case they think some different numerical examples should be included in this software package.

3. Troubleshooting

3.1. Frequently Asked Questions (FAQs)

1) **How do I cite the LAMBDA 4.0 toolbox in my research?**

   Answer: When using this toolbox in your research, please cite it as follows: "Massarweh, L., Verhagen, S., and Teunissen, P.J.G. New LAMBDA toolbox for mixed-integer models: Estimation and Evaluation. GPS Solut NN, XXX (2024), submitted. DOI: *not yet available*"

2) **Can LAMBDA 4.0 be used with older versions of MATLAB?**

   Answer: Yes, this LAMBDA 4.0 toolbox implementation has been developed in MATLAB R2024a version, but also tested for retro compatibility with the R2018b version. If you encounter any compatibility issues, please contact support as soon as possible (see Section 3.2).

3) **How can I contribute to the development of the LAMBDA toolbox?**

   Answer: Contributions to the LAMBDA toolbox are welcome. User can contribute by reporting bugs, suggesting new features, or submitting code fixes. Please contact us for more details on how to effectively contribute.

4) **Do I need to make always use of the main LAMBDA.m and Ps_LAMBDA.m scripts when integrating this toolbox in my software?**

   Answer: No, we provide these main scripts to facilitate the use of LAMBDA 4.0 for non-experienced users. However, many standalone functionalities can be found in the sub-folder 'LAMBDA_toolbox' and they could be used directly within your software. Please, make always sure to reference our publication (Massarweh et al., 2024) when using any of these toolbox functionalities.

5) **More FAQs will be added in future updates of this User Manual (MATLAB).**

3.2. Contacting Support

For any inquiry about the MATLAB implementation, we suggest users to contact us at

LAMBDAtoolbox-CITG-GRS@tudelft.nl

and please, use the following email object: **[LAMBDA-4]**.

References

Massarweh, L., Verhagen, S., and Teunissen, P.J.G. New LAMBDA toolbox for mixed-integer models: Estimation and Evaluation. GPS Solut NN, XXX (2024), submitted. DOI: not yet available.